Sequence & Series...

27 August 2021 05:15 PM

A, D

Q11 → $S_p = a$ $\quad \frac{p}{2}\left(2A + (p-1)D\right) = a$ — (1)

$S_q = b$ $\quad \frac{q}{2}\left(2A + (q-1)D\right) = b$ — (2)

$S_r = c$ $\quad \frac{r}{2}\left(2A + (r-1)D\right) = c$ — (3)

To prove...

LHS

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

from (1), (2) & (3)

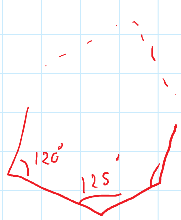
$$\frac{\frac{\cancel{p}}{2}\left(2A+(p-1)D\right)}{\cancel{p}}(q-r) + \frac{\frac{\cancel{q}}{2}\left(2A+(q-1)D\right)(r-p)}{\cancel{q}} + \frac{\frac{\cancel{r}}{2}\left(2A+(r-1)D\right)(p-q)}{\cancel{r}}$$

$$\frac{1}{2}\Big(2A(q-r) + (p-1)(q-r)D$$
$$+ 2A(r-p) + (q-1)(r-p)D$$
$$+ 2A(p-q) + (r-1)(p-q)D\Big)$$

$$\frac{1}{2}\Big(2A(\cancel{q}-\cancel{r}+\cancel{r}-\cancel{p}+\cancel{p}-\cancel{q}) + D\big(\cancel{pq}-\cancel{pr}-\cancel{q}+\cancel{r}$$
$$\cancel{qr}-\cancel{pq}-\cancel{r}+\cancel{p}$$
$$+\cancel{rp}-\cancel{qr}-\cancel{p}+\cancel{q}\big)\Big)$$

$$\frac{1}{2}\left(2A(0) + D(0)\right) = 0 \quad \text{RHS}$$

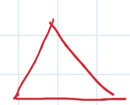
Q18



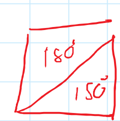
$a = 120°$

$d = 5°$

n ← no of terms in AP

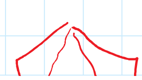


180°



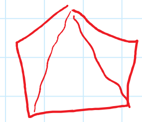
360°

$$(n-2)\times 180° = \frac{n}{2}\left(2(120°) + (n-1)5\right)$$



540°

$$(n-2)\times 180 = \frac{n}{2}\left(2(120)+(n-1)5\right)$$



540°

Sum of all angles in a polygon with n sides $(n-2)\times 180^\circ$

$$(n-2)\,180^\circ = \frac{n}{2}\,(5n+235)$$

$$360n - 720 = 5n^2 + 235n$$

$$5n^2 - 125n + 720 = 0$$

$$n^2 - 25n + 144 = 0$$

$$n = 16, 9$$

$\boxed{n=9}$ $\boxed{n\neq 16}$

—x—x—x—

$>180^\circ$

Largest angle

$a_9 = a + (n-1)d$

$a_9 = 120 + 8(5)$
$= 160^\circ$

$a_{16} = 120^\circ + 15(5)$
$= 120 + 75$
$\Rightarrow 195^\circ > 180^\circ$

(Largest angle is greater than 180°

GP

Ex 9

GP $5, 25, 125 \ldots$

$5^1, 5^2, 5^3 \ldots$

$a = 5$

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = 5$

$a_{10} = 5^{10}$

$a_n = 5^n$

$$a_n = a r^{n-1}$$

$$a_n = 5(5)^{n-1}$$

$\boxed{a_n = 5^n}$ $\boxed{a_{10} = 5^{10}}$

Ex 10 → GP : $2, 8, 32 \ldots\ldots$

$a, ar, ar^2, \ldots$

$a = 2$

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = 4$

$a_n = 131072$

$n = ??$

$n = ??$

4	65536
4	13884
	3471

$$a_n = ar^{n-1}$$

$$\cancel{131072 = 2(4)^{n-1}}$$

$$65536 = 4^{n-1}$$

$$4^8 = 4^{n-1}$$

$$n-1 = 8 \qquad \boxed{n = 9}$$

Ex 11

$a_3 = 24 \Rightarrow ar^2 = 24$ — (1) $\left(a_n = ar^{n-1}\right)$

$a_6 = 192 \Rightarrow ar^5 = 192$ — (2)

(2) $\div$ (1) $\dfrac{\cancel{a}r^5}{\cancel{a}r^2} = \dfrac{192}{24}$

$$r^3 = 8$$

$$\boxed{r = 2} \text{ — (3)}$$

from (1) & (3)

$$a(2^2) = 24$$

$$\boxed{a = 6} \text{ — (4)}$$

$$a_{10} = ar^9$$

$$= (6)(2)^9$$

$$= 6 \times 512$$

$$\Rightarrow 1024 \times 3$$

$$\boxed{a_{10} = 3072}$$

Ex 13 →

GP $= 3, \frac{3}{2}, \frac{3}{4}, \ldots\ldots\ldots$

$a = 3$

$r = \frac{1}{2}$

$S_n = \dfrac{a(r^n - 1)}{(r-1)}, \; r > 1$

$S_n = \dfrac{3069}{512}$

$n = ??$

$$S_n = \frac{a(1 - r^n)}{(1-r)}$$

$$\frac{3069}{512} = S_n = \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$\frac{\cancel{3069}^{\,1023}}{512} = \frac{\cancel{3}\left(1-\left(\frac{1}{2}\right)^n\right)}{\frac{1}{2}}$$

$$\frac{1023}{1024} = 1 - \frac{1}{2^n}$$

$$\frac{1}{2^n} = 1 - \frac{1023}{1024}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$$2^n = 1024$$

$$2^n = 2^{10}$$

$$\boxed{n = 10}$$

—x—x—x—x—

Ex 12

GP : $1, \frac{2}{3}, \frac{4}{9}, \ldots$

$a = 1$

$r = \frac{2}{3}$

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{1\left(1-\left(\frac{2}{3}\right)^n\right)}{\left(1-\frac{2}{3}\right)} = \frac{\left(1-\left(\frac{2}{3}\right)^n\right)}{\left(\frac{1}{3}\right)} = 3\left(1-\left(\frac{2}{3}\right)^n\right)$$

$$S_5 = 3\left(1-\left(\frac{2}{3}\right)^5\right) = 3\left(1-\frac{32}{243}\right) = \cancel{3}\left(\frac{211}{\cancel{243}_{\,81}}\right) = \frac{211}{81}$$

NOTE →

If 3 terms are in AP

$a-d, \ a, \ a+d$

$A = a-d$

$D = d$

If 4 term in AP

$a-3d, \ a-d, \ a+d, \ a+3d$

$A = a-3d$

$D = 2d$

If 5 terms are in AP

$a-2d,\ a-d,\ a,\ a+d,\ a+2d$ $\qquad A = a-2d$, $D = d$

If 3 terms are in GP

$\frac{a}{r},\ a,\ ar$ $\qquad A = \frac{a}{r}$, $R = r$

If 4 terms are in GP $\frac{a}{r^3},\ \frac{a}{r},\ ar,\ ar^3$ $\qquad A = \frac{a}{r^3}$, $R = r^2$

Ex14 Lets say 3 terms in GP be $\frac{a}{r}, a, ar$ $\qquad \boxed{A = \frac{a}{r},\ R = r}$

$\frac{a}{r} + a + ar = \frac{13}{12}$ $\qquad \frac{a}{\cancel{r}} \times a \times a\cancel{r} = -1$

$a^3 = -1$

$\boxed{a = -1}$ — ①

from ①

$-\frac{1}{r} - 1 - r = \frac{13}{12}$

$\frac{1 + r + r^2}{r} = -\frac{13}{12}$

$12r^2 + 12r + 12 = -13r$

$12r^2 + 25r + 12 = 0$

$12r^2 + 16r + 9r + 12 = 0$

$4r(3r+4) + 3(3r+4)$

$(4r+3)(3r+4)$

$r = -\frac{3}{4}$ or $-\frac{4}{3}$

—x—x—x—

For $a = -1$, $r = -\frac{3}{4}$

Terms are $\frac{a}{r} = \frac{4}{3}$, $a = -1$, $ar = \frac{3}{4}$

$\boxed{\frac{4}{3}, -1, \frac{3}{4}}$

for $a = -1$, $r = -\frac{4}{3}$

$\frac{a}{r} = \frac{3}{4}$ $\quad a = -1$ $\quad ar = \frac{4}{3}$

$\boxed{\frac{3}{4}, -1, \frac{4}{3}}$

— x — x — x — x —

Ex15 ⇒ $7, 77, 777, \ldots\ldots$

$S_n = 7 + 77 + 777 + \ldots\ldots + \underbrace{77\ldots7}_{n \text{ times}}$

$S_n = 7\,(1 + 11 + 111 + \ldots\ldots + \underbrace{11\ldots11}_{n \text{ times}})$

$$S_n = \frac{7}{9}\left(9 + 99 + 999 + \cdots \underbrace{99\ldots99}_{n\text{ times}}\right)$$

$$S_n = \frac{7}{9}\left(10-1 + 100-1 + 1000-1 \cdots + \underbrace{1000\ldots0}_{n\text{ zeroes}} - 1\right)$$

$$S_n = \frac{7}{9}\left(\underbrace{10 + 10^2 + \cdots + 10^n}_{GP} - \left(\underbrace{1+1+1\cdots+1}_{n\text{ times}}\right)\right)$$

$a=10$, $r=10$, n

$$S_n = \frac{7}{9}\left(\frac{10(10^n-1)}{10-1} - n\right) = \frac{7}{9}\left(\frac{10}{9}(10^n-1) - n\right)$$

—x—x—x—x—

$a, b, c \Rightarrow AP$

$\Downarrow GP$

$\frac{b}{a} = \frac{c}{b}$

$b^2 = ac$

$\boxed{b = \sqrt{ac}}$

$b - a = c - b$

$2b = a + c$

$\boxed{b = \frac{a+c}{2}}$

$$\boxed{\begin{aligned} AM &= \frac{a+b}{2} \\ GM &= \sqrt{ab} \end{aligned}}$$

Let 2 numbers be a & b ....

Ex18 → $AM = 10$ $\quad GM = 8$

$\frac{a+b}{2} = 10$ $\quad \sqrt{ab} = 8$

$a + b = 20$ —① $\quad ab = 64$ —②

M1

from ① & ②

$a + \frac{64}{a} = 20$

$a^2 + 64 = 20a$

$a^2 - 20a + 64 = 0$

$a^2 - 16a - 4a + 64 = 0$

$a = 4$ or 16

If $a = 4$ & If $a = 16$

M2

$(a+b)^2 - 4ab = (a-b)^2$

$20^2 - 4(64) = (a-b)^2$

$400 - 256 = (a-b)^2$

$144 = (a-b)^2$

$a - b = \pm 12$ —③

—✓—✓—✓—

$a + b = 20$
$a - b = 12$

$a + b = 20$
$a - b = -12$

If $a=4$, $b=16$ & If $a=16$, $b=4$

$a-b=12$

$a=16,\ b=4$

$a-b=-12$

$a=4,\ b=16$

Exercise 9.3

Q3 →

$a_5 = p \implies ar^4 = p$

$a_8 = q \implies ar^7 = q$

$a_{11} = S \implies ar^{10} = S$

LHS $= q^2$

$= (ar^7)^2$

$= a^2 r^{14}$

RHS $= pS$

$= (ar^4)(ar^{10})$

$= a^2 r^{14}$

so $\boxed{q^2 = pS}$ Hence proved . . .

Q4 →

$a_4 = (a_2)^2$

$ar^3 = (ar)^2$

$\cancel{a} r^{\cancel{3}} = a^{\cancel{2}} \cancel{r^2}$

$\boxed{a = r}$

$a_4 = ar^3$

$a_2 = ar$

$a = -3$ — ①

Given

$a = -3$

$r = -3$

GP

$-3,\ 9,\ -27,\ 81, \ldots$

$a_7 = ar^6$

$a_7 = (-3)(-3)^6$

$a_7 = (-3)(729)$

$\boxed{a_7 = -2187}$

—x—x—x—x—

Q8 →

$\sqrt{7},\ \sqrt{21},\ 3\sqrt{7}\ \ldots$ n terms

$\frac{a_2}{a_1} = \sqrt{3}$

$\frac{a_3}{a_2} = \frac{3\cancel{\sqrt{7}}}{\cancel{\sqrt{21}}\sqrt{3}} = \sqrt{3}$

$a = \sqrt{7}$
$r = \sqrt{3}$

$$S_n = \frac{a(r^n - 1)}{(r-1)} = \frac{\sqrt{7}\left((\sqrt{3})^n - 1\right)}{(\sqrt{3}-1)}$$

$$\Rightarrow \frac{\sqrt{7}(3^{\frac{n}{2}} - 1)}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \frac{\sqrt{7}}{2}(3^{n/2} - 1)(\sqrt{3}+1)$$

Q11 →

$$\sum_{k=1}^{11} (2 + 3^k)$$

$$\boxed{\sum(f_1 \pm f_2) = \sum f_1 \pm \sum f_2}$$

$$\Rightarrow \sum_{k=1}^{11} 2 + \sum_{k=1}^{11} 3^k$$

$$(2+3^1) + (2+3^2) + (2+3^3) \cdots + (2+3^{11})$$

$$\underbrace{(2+2+\cdots 2)}_{11 \text{ terms}} + \underbrace{(3^1 + 3^2 + \cdots 3^{11})}_{11 \text{ terms}}$$

$a = 3$
$r = 3$

$$S_n = a\left(\frac{r^n - 1}{r-1}\right)$$

$$2(11) + 3\left(\frac{3^{11}-1}{3-1}\right)$$

$$22 + \frac{3}{2}(3^{11} - 1)$$

—x—x—x—x—

Q13 - $3, 3^2, 3^3 \cdots$

$a = 3, \quad r = 3$

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

$S_n = 120$

$$= \frac{\cancel{3}(3^n - 1)}{(3-1)} = \cancel{120}\ 40$$

$$\Rightarrow 3^n - 1 = 80$$

$$3^n = 81$$

$$3^n = 3^4$$

$$\boxed{n = 4}$$

—x—x—x—

Q14 → Sum of first 3 terms

$$a + ar + ar^2 = 16 \qquad a(1 + r + r^2) = 16 \quad -(1)$$

Sum of next 3 terms

$ar^3 + ar^4 + ar^5 = 128$ (2) $\qquad r^3 a(1+r+r^2) = 128$ (2)

(2) $\div$ 1

$r^3 = 8$

$\boxed{r = 2}$ (3)

from (1) & (3)

$a(1+2+4) = 16$

$\boxed{a = \frac{16}{7}}$

$S_n = a\left(\frac{r^n - 1}{r-1}\right)$

$S_n = \frac{16}{7}\left(\frac{2^n - 1}{2-1}\right)$

$\boxed{S_n \Rightarrow \frac{16}{7}(2^n - 1)}$

—x—x—x—

Q19 →

$S_1 = 2, 4, 8, 16, 32, \ldots\ldots$

$S_2 = 128, 32, 8, 2, \frac{1}{2}, \ldots\ldots$

Product of corresponding terms $= 256, 128, 64, 32, 16.$

$a = 256$

$r = \frac{1}{2}$

$n = 5$

$S_n = a\frac{(1 - r^n)}{(1-r)}$

$S_5 = 256\left(\frac{1 - (\frac{1}{2})^5}{1 - \frac{1}{2}}\right)$

$S_5 = \cancel{256}^{16} \times 2 \left(\frac{31}{\cancel{32}_{\cancel{16}}}\right)$

—x—x—x—

$S_5 = 16 \times 31$

$S_5 = 496$

Q20 → $S_1 = a, ar, ar^2, ar^3 \ldots\ldots$

$S_2 = A, AR, AR^2, AR^3, \ldots\ldots$

Product of corresponding terms $= aA, aArR, aA(rR)^2, aA(rR)^3 \ldots\ldots \Rightarrow GP$

First term $\Rightarrow aA$

Common Ratio $= rR$

—x—x—

terms

Common Ratio = ~~R~~ r K

Q21 →

Let 4 numbers in GP be

First term 'a'

Common Ratio 'r'

a, ar, ar^2, ar^3

$a_3 = a_1 + 9$

$\boxed{ar^2 = a + 9}$

$a(r^2-1) = 9$ — (1)

$a_2 = a_4 + 18$

$\boxed{ar = ar^3 + 18}$

$ar(1-r^2) = 18$

$-r(a(r^2-1)) = 18$ — (2)

Put (1) in (2)

$-r(9) = 18$

$\boxed{r = -2}$ — (3)

Put (3) in (1)

$a(4-1) = 9$

$\boxed{a = 3}$ (4)

Terms are

3, −6, 12, −24

Q22 →

First term = A

Common Ratio = R

$a_p = a \Rightarrow AR^{p-1} = a$ — (1)

$a_q = b \Rightarrow AR^{q-1} = b$ — (2)

$a_r = c \Rightarrow AR^{r-1} = c$ — (3)

LHS = $a^{q-r}\, b^{r-p}\, c^{p-q}$

from (1),(2) & (3)

$$(AR^{p-1})^{(q-r)} (AR^{q-1})^{(r-p)} (AR^{r-1})^{(p-q)}$$

$$A^{(q-r)+(r-p)+(p-q)}\, R^{(pq-pr-q+r)+(qr-qp-r+p)+(pr-qr-p+q)}$$

$$A^0\, R^0$$

$$1 \times 1$$

$1 = \underline{RHS}$